FALL 2014 MATH 24 LINEAR ALGEBRA
MIDTERM EXAM I OCTOBER 16 2014

Name:____

Instructions:

(1) There are 10 multiple-choice problems (2 points each) and each problem has 5 choices, a subset of which is correct. Such subset could be empty.
(2) No books, notes or electronic devices are allowed.
(3) Write your answers to the left of each problem. If your answer is empty, leave blank or write $\emptyset$.
(4) All notations are standard. Unless specified, $\mathbb{R}^n$, $\mathcal{F}(\mathbb{R})$, $M_{n\times m}(\mathbb{R})$, $\mathcal{L}(V,W)$ and their subspaces always have standard addition and scalar multiplication. Unless specified the field of scalars is always $\mathbb{R}$. All matrix operations assume that the matrices are of appropriate sizes for the operation to be valid.
(5) Problems are *not* ordered by their difficulty.
(6) If your answer is correct, then you receive 2 points; if the difference between the correct answer and your answer is 1, then you receive 1 point; otherwise you receive 0.
(7) You can use the last page as scratch.

Problem 1. *Which of the following are vector spaces?*
(A) $\mathbb{R}^2$, *where the operations are defined as* $(x,y)+(z,w)=(x+z,y+w)$, $c\cdot(x,y)=(cx,-cy)$;
(B) All integer numbers over the field F_2, *where the addition is defined naturally and scalar multiplication is defined by* $0\cdot x=0$, $1\cdot x=x$;
(C) $\{f\in\mathcal{D}(\mathbb{R}) : f'+f=0\}$;
(D) $\{cat, dog\}$ *over* F_2, *where* $cat+cat=dog+dog=cat$, $cat+dog=dog+cat=dog$, $0\cdot cat=cat$, $0\cdot dog=cat$, $1\cdot cat=dog$, $1\cdot dog=dog$.
(E) All 2×2 *matrices whose rank (i.e., the rank of the corresponding left multiplication transformation) is less than or equal to* 1.

Problem 2. *Which of the following equations define subspaces of* $\mathbb{R}^3=\{(x,y,z) : x,y,z\in\mathbb{R}\}$*?*
(A) $xy+yz+zx=0$;
(B) $3x+4y+5=0$;
(C) $z=0$;
(D) $x^2+y^2+z^2=0$;
(E) $xyz=0$.

Problem 3. *Which of the following are linear transformations?*
(A) $T:\mathbb{R}^2\to\mathbb{R}^2$, $T(x,y)=(x+y,xy)$;
(B) $T:\mathbb{R}^2\to\mathcal{F}(\mathbb{R})$, $T(a,b)(x)=ax+b$;
(C) $T:\mathbb{R}^3\to\mathcal{F}(\mathbb{R})$, $T(a,b,c)(x)=ae^x+bx+c$;
(D) $T:\mathcal{F}(\mathbb{R})\to\mathcal{F}(\mathbb{R})$, $T(f)(x)=f(x+1)$;
(E) $T:\mathcal{F}(\mathbb{R})\to\mathbb{R}^2$, $T(f)=(f(0),f(1))$.

Problem 4. *Let* $T:V\to W$ *be linear and* $\dim(V)=3$, $\dim(W)=5$. *Which of the following statements are true?*
(A) T *cannot be surjective;*
(B) If T *is injective, then* $null(T)=3$;
(C) If $rank(T)=2$, *then* $null(T)=1$;
(D) If $null(T)=2$, *then* $rank(T)=3$;
(E) If T *is the zero map, then* $null(T)=3$.

Problem 5. *Which of the following linear transformations are invertible?*
(A) $T:\mathbb{R}^2\to\mathbb{R}^2$, $T(a,b)=(a+b,a-b)$;
(B) $T:\mathcal{F}(\mathbb{R})\to\mathcal{F}(\mathbb{R})$, $T(f)(x)=2f(2x)$;
(C) $T:\mathcal{L}(\mathbb{R})\to\mathbb{R}$, $T(f)=f(\sqrt{2})$;
(D) $T:\mathcal{P}(\mathbb{R})\to\mathcal{P}(\mathbb{R})$, $T(f)=2f+f'$;
(E) $T:\mathcal{M}_{3\times3}(\mathbb{R})\to\mathcal{M}_{3\times3}(\mathbb{R})$, $T(A)=A^t+A$.

Problem 6. *Which of the following matrices represent an onto (i.e., surjective) linear transformation?*

(A) $\begin{pmatrix}1&1\\1&1\end{pmatrix}$; *(B)* $\begin{pmatrix}-4&3\\3&4\end{pmatrix}$; *(C)* $\begin{pmatrix}1&1\end{pmatrix}$; *(D)* $\begin{pmatrix}3&1\\0&1\\2&2\end{pmatrix}$; *(E)* $\begin{pmatrix}3&1&2\\1&1&1\\-1&1&0\end{pmatrix}$

Problem 7. *Which of the following matrices are invertible?*

(A) (5); *(B)* $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$; *(C)* $\begin{pmatrix} 1 & 3 & 4 \\ 1 & 1 & 2 \\ 2 & 0 & 3 \end{pmatrix}$; *(D)* $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$; *(E)* $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$.

Problem 8. *Which of the following β are bases for the corresponding vector spaces?*
(A) $V = \mathcal{P}_2(\mathbb{R})$, $\beta = \{x^2 + x + 1, x + 1, x\}$;
(B) $V = \mathbb{R}$, $\beta = \{\pi\}$;
(C) $V = \mathbb{C}$, $\beta = \{1 + i, 1 - i\}$;
(D) $V = M_{2\times 2}(\mathbb{R})$, $\beta = \{\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\}$;
(E) $V = \mathcal{L}(\mathbb{R})$, $\beta = \{f\}$ *where* $f(x) = \pi x$.

Problem 9. *Which of the following collections of vectors are linearly independent (with respect to natural definitions of addition and scalar multiplication)?*
(A) $1, \pi$ *in* $\mathbb{C}$;
(B) $(1, 1, 2)$, $(-1, 2, 3)$, $(1, 4, 7)$ *in* $\mathbb{R}^3$;
(C) $x^2 + x + 1$, $x^2 - 1$, $x + 1$ *in* $\mathcal{F}(\mathbb{R})$;
(D) x^3, x, 1 *in* $\mathcal{P}(F_3)$;
(E) I_V, 0_V *(identity map and zero map) in* $\mathcal{L}(V)$.

Problem 10. *Which of the following statements are in general* **false***?*
(A) $(AB)^t = B^t A^t$;
(B) $(A^t)^{-1} = (A^{-1})^t$ *if* A *is invertible;*
(C) $(A + B)^t = A^t + B^t$;
(D) $AB^t = (BA^t)^t$;
(E) $A^{-1} = ((A^t)^{-1})^t$ *if* A *is invertible.*